Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Computational and Applied Mathematics 188 (2006) 256–264
JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS
www.elsevier.com/locate/cam

Periodic solutions of a nonautonomous periodic model of population with continuous and discrete time

Yongli Song[a,*], Yahong Peng[b]

[a] *Department of Applied Mathematics, Tongji University, Shanghai 200092, PR China*
[b] *Department of Applied Mathematics, Donghua University, Shanghai 200051, PR China*

Received 25 March 2003; received in revised form 30 March 2005

Abstract

In this paper, we employ the Mawhin's continuation theorem to study the existence of positive periodic solutions of the nonautonomous periodic model of population with continuous and discrete time. It is interesting that the conditions to guarantee the existence of positive periodic solutions of discrete time model are similar to those for the corresponding continuous time model.

Keywords: Periodic solution; Continuous time; Discrete time; Coincidence degree

1. Introduction

In order to describe the control of a single population of cells, Nazarenko [7] proposed the following nonlinear delay differential equation:

$$x'(t) + px(t) - \frac{qx(t)}{r + x^n(t-\tau)} = 0, \tag{1.1}$$

where $p, q, r, \tau \in (0, +\infty), n \in \mathbb{N} = \{1, 2, \ldots\}$ and $q/p > r$. For a long time, Eq. (1.1) has been well studied by many authors (see [5,8,10] and their references). Recently, according to the periodic

* Corresponding author. Tel.:/fax: +86 2154747534.
E-mail address: syl.mail@163.com (Y. Song).

0377-0427/$ - see front matter
doi:10.1016/j.cam.2005.04.017

assumption of the environmental variation, Saker [9] proposed the following nonautonomous delay differential equation:

$$x'(t) + p(t)x(t) - \frac{q(t)x(t)}{r + x^n(t - m\omega)} = 0, \tag{1.2}$$

where m and n are positive integers, and $p(t)$ and $q(t)$ are positive periodic functions of period ω.

In the present paper, we will consider the more general modification of Eq. (1.1):

$$y'(t) + p(t)y(t) - \frac{q(t)y(t)}{r + y^n(t - \tau(t, y(t)))} = 0, \tag{1.3}$$

where $p, q \in C(\mathbb{R}, \mathbb{R}^+)$ and $\tau \in C(\mathbb{R}^2, \mathbb{R}^+)$, $\mathbb{R}^+ = [0, +\infty)$, are ω periodic with respect to t, and r is a constant.

By Eq. (1.3), it is easy to obtain

$$y(t) = y(0) \exp\left\{ \int_0^t \left[-p(t) + \frac{q(t)}{r + y^n(t - \tau(t, y(t)))} \right] \mathrm{d}s \right\}.$$

It follows that $\mathbb{R}_+ = \{y | y > 0\}$ is invariant with respect to Eq. (1.3). Considering the biological significance, it suffices to consider only positive solutions of Eq. (1.3).

The paper is organized as follows. In Section 2, we apply the Mawhin's continuation theorem to investigate the existence of a positive periodic solution of Eq. (1.3). In Section 3, with the help of the differential equations with piecewise constant arguments, we first propose a discrete analogue of Eq. (1.3) without delay, and then investigate its positive periodic solutions.

2. Existence of positive periodic solutions of Eq. (1.3)

In order to obtain the existence of positive periodic solutions of Eq. (1.3), for the reader's convenience, we shall summarize a few concepts and results form [4].

Let X, Y be normed vector spaces, let $L : \mathrm{Dom}\, L \subset X \to Y$ be a linear mapping, and let $\mathbb{N} : X \to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim \mathrm{Ker}\, L = \mathrm{codim}\, \mathrm{Im}\, L < +\infty$ and $\mathrm{Im}\, L$ is closed in Y. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\mathrm{Im}\, P = \mathrm{Ker}\, L$ and $\mathrm{Im}\, L = \mathrm{Ker}\, Q = \mathrm{Im}\,(I - Q)$. It follows that $L|\mathrm{Dom}\, L \cap \mathrm{Ker}\, P : (I - P)X \to \mathrm{Im}\, L$ is invertible. We denote the inverse of that map by K_p. If Ω is an open bounded subset of X, the mapping $\mathbb{N}$ will be called L-compact on $\overline{\Omega}$ if $Q\mathbb{N}(\overline{\Omega})$ is bounded and $K_p(I - Q)\mathbb{N} : \overline{\Omega} \to X$ is compact. Since $\mathrm{Im}\, Q$ is isomorphic to $\mathrm{Ker}\, L$, there exists an isomorphism $J : \mathrm{Im}\, Q \to \mathrm{Ker}\, L$.

In the proof of our existence theorem, we will use the continuation theorem [4] as follows.

Lemma 2.1 (*Continuation theorem*). *Let L be a Fredholm mapping of index zero and $\mathbb{N}$ be L-compact on $\overline{\Omega}$. Furthermore suppose*:

(a) *for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \mathrm{Dom}\, L$,*

$$Lx \neq \lambda \mathbb{N} x$$

is such that;

(b) $Q\mathbb{N}x \neq 0$ *for each* $x \in \partial\Omega \cap \operatorname{Ker} L$ *and the Brouwer degree*

$$\deg\{JQ\mathbb{N}, \Omega \cap \operatorname{Ker} L, 0\} \neq 0.$$

Then the operator equation $Lx = \mathbb{N}x$ *has at least one solution lying in* $\operatorname{Dom} L \cap \overline{\Omega}$.

For a continuous and periodic function $f(t)$ with period ω, denote by $\bar{f}$ the average of $f(t)$ over an interval of length ω, i.e.,

$$\bar{f} = \frac{1}{\omega}\int_0^\omega f(t).$$

Our main result on the existence of a periodic solution of Eq. (1.3) is stated in the following theorem.

Theorem 2.1. *If* $\overline{q}/\overline{p} > r$, *then Eq.* (1.3) *has at least one positive periodic solution with period* ω.

Proof. Make the change of variable

$$y(t) = \exp\{x(t)\}.$$

Then Eq. (1.3) can be written as the following form:

$$x'(t) = -p(t) + \frac{q(t)}{r + \exp\{nx(t - \tau(t, \exp\{x(t)\}))\}}. \tag{2.1}$$

Clearly, if Eq. (2.1) has a periodic solution $x(t)$, then Eq. (1.3) has a positive periodic solution $y(t) = \exp\{x(t)\}$. Thus, it is sufficient to prove that Eq. (2.1) has at least one periodic solution.

To apply Lemma 2.1 to Eq. (2.1), we first define

$$X = Y = \{x(t) \in C(\mathbb{R}, \mathbb{R}) : x(t + \omega) = x(t)\}$$

and $\|x\| = \max_{t\in[0,\omega]}|x(t)|$ for any $x \in X$ (or Y). Then X and Y are Banach spaces when they are endowed with the norm $\|\cdot\|$.

Let

$$L : \operatorname{Dom} L \cap X \to X, \quad Lx = x'(t), \quad \operatorname{Dom} L = \{x(t) \in X : x(t) \in C^1(\mathbb{R}, \mathbb{R})\}$$

and

$$\mathbb{N} : X \to X, \quad \mathbb{N}x = -p(t) + \frac{q(t)}{r + \exp\{nx(t - \tau(t, \exp\{x(t)\}))\}}.$$

Define projectors P and Q as

$$Px = Qx = \frac{1}{\omega}\int_0^\omega x(t)\,\mathrm{d}t, \quad x \in X \text{ (or } Y).$$

Clearly, $\operatorname{Ker} L = \mathbb{R}$, and $\operatorname{Im} L = \{x \in Y;\ \int_0^\omega x(t) = 0\}$ is closed in Y and

$$\dim \operatorname{Ker} = \operatorname{codim} \operatorname{Im} L = 1.$$

Therefore, L is Fredholm mapping of index 0. In addition, it is easy to verify that P, Q are continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q).$$

Straightforward computation shows that the inverse (to L_p) $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ has the form

$$K_p(x) = \int_0^t x(s)\,\mathrm{d}s - \frac{1}{\omega}\int_0^\omega \int_0^t x(s)\,\mathrm{d}s\,\mathrm{d}t, \qquad t \in [0, \omega].$$

Thus,

$$Q\mathbb{N}x = \frac{1}{\omega}\int_0^\omega \left[-p(s) + \frac{q(s)}{r + \exp\{nx(s - \tau(s, \exp\{x(s)\}))\}}\right]\mathrm{d}s.$$

$$\begin{aligned} K_p(I - Q)\mathbb{N}x = {} & \int_0^t \left[-p(s) + \frac{q(s)}{r + \exp\{nx(s - \tau(s, \exp\{x(s)\}))\}}\right]\mathrm{d}s \\ & - \frac{1}{\omega}\int_0^\omega \int_0^t \left[-p(s) + \frac{q(s)}{r + \exp\{nx(s - \tau(s, \exp\{x(s)\}))\}}\right]\mathrm{d}s\,\mathrm{d}t \\ & - \left(\frac{t}{\omega} - \frac{1}{2}\right)\int_0^\omega \left[-p(s) + \frac{q(s)}{r + \exp\{nx(s - \tau(s, \exp\{x(s)\}))\}}\right]\mathrm{d}s. \end{aligned}$$

Notice that $Q\mathbb{N}$ and $K_p(I - Q)\mathbb{N}$ are continuous by the Lebesgue theorem, and $Q\mathbb{N}(\overline{\Omega})$ and $K_p(I - Q)\mathbb{N}(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Thus, $\mathbb{N}$ is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$. We now reach the position to search for an appropriate open bounded subset for the application of the continuation theorem.

Consider the equation $Lx = \lambda\mathbb{N}x$, $\lambda \in (0, 1)$, which is

$$x'(t) = \lambda\left[-p(t) + \frac{q(t)}{r + \exp\{nx[t - \tau(t, \exp\{x(t)\})]\}}\right], \qquad \lambda \in (0, 1). \tag{2.2}$$

Suppose that $x(t) \in X$ is an arbitrary solution of Eq. (2.2) for a certain $\lambda \in (0, 1)$. Integrating on both sides of Eq. (2.2) over the interval $[0, \omega]$, we have

$$0 = \int_0^\omega \left[-p(t) + \frac{q(t)}{r + \exp\{nx(t - \tau(t, \exp\{x(t)\}))\}}\right]\mathrm{d}t,$$

which leads to

$$\int_0^\omega \left[\frac{q(t)}{r + \exp\{nx(t - \tau(t, \exp\{x(t)\}))\}}\right]\mathrm{d}t = \overline{p}\omega. \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$\int_0^\omega |x'(t)|\,\mathrm{d}t \leqslant \int_0^\omega p(t)\,\mathrm{d}t + \int_0^\omega \frac{q(t)}{r + \exp\{nx(t - \tau(t, \exp\{x(t)\}))\}}\,\mathrm{d}t = 2\overline{p}\omega. \tag{2.4}$$

Since $x(t) \in X$, there exist $\xi, \eta \in [0, \omega]$ such that

$$x(\xi) = \max_{t \in [0,\omega]} x(t) \quad \text{and} \quad x(\eta) = \min_{t \in [0,\omega]} x(t). \tag{2.5}$$

Thus, from (2.3) and (2.5), we obtain that

$$\overline{p}\omega \leqslant \int_0^{\omega} \frac{q(t)}{r+\exp\{nx(\eta)\}}\,\mathrm{d}t = \frac{\overline{q}\omega}{r+\exp\{nx(\eta)\}},$$

which reduces to

$$x(\eta) \leqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}. \tag{2.6}$$

On the other hand, it follows from (2.3) and (2.5) that

$$\overline{p}\omega \geqslant \int_0^{\omega} \frac{q(t)}{r+\exp\{nx(\xi)\}}\,\mathrm{d}t = \frac{\overline{q}\omega}{r+\exp\{x(\xi)\}},$$

which leads to

$$x(\xi) \geqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}. \tag{2.7}$$

From (2.6) and (2.7), it is easy to see that there exists a $t_0 \in [0, \omega]$ such that

$$x(t_0) = \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}. \tag{2.8}$$

Therefore, from (2.4) and (2.8), we obtain that

$$|x(t)| \leqslant |x(t_0)| + \left|\int_{t_0}^{t} x'(t)\,\mathrm{d}t\right| \leqslant |x(t_0)| + \int_0^{\omega} |x'(t)|\,\mathrm{d}t \leqslant \frac{1}{n}\left|\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}\right| + 2\overline{p}\omega = B_1.$$

Clearly, B_1 is independent of λ. Take $B = B_1 + B_2$, where B_2 is taken sufficiently large such that $B_2 > |\frac{1}{n}\ln\{\frac{\overline{q}}{\overline{p}} - r\}|$. We now let $\Omega = \{x(t) \in X : \|x\| < B\}$. It is clear that Ω verifies requirement (a) in Lemma 2.1. If $x \in \partial\Omega \cap \operatorname{Ker} L$, then x is a constant with $\|x\| = B$. By the definition of B, it is easy to verify that

$$Q\mathbb{N}x = -\overline{p}\omega + \frac{\overline{q}\omega}{r+\exp\{nx\}} \neq 0.$$

Furthermore, it is obvious that

$$\deg\{JQ\mathbb{N}x, \Omega \cap \operatorname{Ker} L, 0\} \neq 0,$$

where J can be the identity mapping since $\operatorname{Im} P = \operatorname{Ker} L$. By now we have proved that Ω verifies all the requirements in Lemma 2.1. Hence, Eq. (2.1) has at least one ω-periodic solution $x^*(t)$. Further, Eq. (1.3) has at least one positive ω-periodic solution $y(t) = \exp\{x^*(t)\}$. The proof is complete. □

3. Discrete analogue of Eq. (1.3) without delay

In recent years, many authors [1,6] have argued that the discrete time models governed by difference equations are more appropriate than the continuous ones, particularly, when the populations have nonoverlapping generations. Such discrete time analogues for population modeled have been done by several authors (see, for example, [2,3,11]). Thanks to differential equations with piecewise constant arguments, we now consider the following version of (1.3) without delay:

$$\frac{1}{y(t)}\frac{\mathrm{d}y(t)}{\mathrm{d}t} = -p([t]) + \frac{q([t])}{r + y^n([t])}, \quad t \neq 0, 1, 2, \ldots. \tag{3.1}$$

where $[t]$ denotes the integer part of t, $t \in (0, +\infty)$. Equation of type (3.1) is known as differential equation with piecewise constant arguments and these equations occupy a position midmay between differential and difference equations.

By a solution of Eq. (3.1), we mean a function $y(t)$, which is defined for $t \in [0, +\infty)$, and possesses the following properties:

(i) $y(t)$ is continuous on $[0, +\infty)$.
(ii) The derivative $\mathrm{d}y(t)/\mathrm{d}t$ exists at each point $t \in [0, +\infty)$ with the possible exception of the point $t \in \{0, 1, 2, \ldots\}$, where left-side derivatives exist.
(iii) The equations in Eq. (3.1) are satisfied on each internal $[k, k+1)$ with $k = 0, 1, 2, \ldots$.

On any interval $[k, k+1)$, $k = 0, 1, 2, \ldots$, we can integrate Eq. (3.1) and obtain for $k \leqslant t < k+1$, $k = 0, 1, 2, \ldots$,

$$y(t) = y(k)\exp\left\{\left(-p(k) + \frac{q(k)}{r + y^n(k)}\right)(t-k)\right\}.$$

Letting $t \to k+1$, we obtain that

$$y(k+1) = y(k)\exp\left\{-p(k) + \frac{q(k)}{r + y^n(k)}\right\}, \tag{3.2}$$

which is a discrete time analogue of Eq. (1.3) without delay. Our main purpose in this section is to investigate the existence of periodic solutions of nonautonomous difference Eq. (3.2). Our method employed in this section is still continuation theorem in Section 2.

Let $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{R}$ and $\mathbb{R}^+$ denote the sets of all integers, nonnegative integers, real numbers, nonnegative real numbers, respectively. For convenience, in what follows, we will use the notation below throughout this section:

$$I_\omega = \{0, 1, \ldots, \omega - 1\}, \quad \overline{f} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} f(k), \quad f(\xi) = \max_{k\in I_\omega}\{x(k)\}, \quad f(\eta) = \min_{k\in I_\omega}\{x(k)\}, \tag{3.3}$$

where $\{f(k)\}$ is an ω-periodic sequence of real numbers defined for $k \in \mathbb{Z}$, i.e., $f(k+\omega) = f(k)$.

In Eq. (3.2), we always assume that r is a positive constant, $p(k), q(k) : \mathbb{Z} \to \mathbb{R}^+$ are ω periodic, where ω, a fixed positive integer, denotes the periodicity of the parameters.

Lemma 3.1 (*Fan and Wang [3]*). *Let $f : \mathbb{Z} \to \mathbb{R}$ be ω-periodic, i.e., $f(k+\omega) = f(k)$, then for any fixed $k_1, k_2 \in I_\omega$ and any $k \in \mathbb{Z}$,*

$$f(k) \leqslant f(k_1) + \sum_{s=0}^{\omega-1} |f(s+1) - f(s)|, \quad f(k) \geqslant f(k_2) - \sum_{s=0}^{\omega-1} |f(s+1) - f(s)|.$$

We are now in the position to state and prove the main result of this section. It is interesting that the result established below for the discrete time Eq. (3.2) is similar as that for the continuous time Eq. (1.3).

Theorem 3.1. *Suppose that* $\overline{q}/\overline{p} > r$. *Then* (3.2) *has at least one positive periodic solution with period* ω.

Proof. Define

$$l = \{y = \{y(k)\} : y(k) \in \mathbb{R}, k \in \mathbb{Z}\}.$$

For $y = y(k) \in \mathbb{R}$, define $\|y\| = \max_{0 \leqslant k \leqslant \omega-1}\{|y(k)|\}$. Let $l^{\omega} \subseteq l$ denote the subspace of all ω-periodic sequences equipped with the usual supremum norm $\|\cdot\|$ for $y \in l^{\omega}$. It is easy to verify that l^{ω} is a finite-dimensional Banach space [3].

Let

$$l_0^{\omega} = \left\{ y = \{y(k)\} \in l^{\omega} : \sum_{k=0}^{\omega-1} y(k) = 0 \right\}$$

and

$$l_c^{\omega} = \{y = \{y(k)\} \in l^{\omega} : y(k) = h \in \mathbb{R}, k \in \mathbb{Z}\},$$

then it follows that l_0^{ω} and l_c^{ω} both are closed linear subspaces of l^{ω} and

$$l^{\omega} = l_0^{\omega} \oplus l_c^{\omega}, \quad \dim\, l_c^{\omega} = 1.$$

It is obvious that the solutions of (3.2) with initial condition $y(0) > 0$ remains positive for all $k \in \mathbb{Z}^+$. Thus, we can introduce a change of variable by the formula $y(k) = \exp\{x(k)\}$ so that Eq. (3.2) can be written as

$$x(k+1) - x(k) = -p(k) + \frac{q(k)}{r + \exp\{nx(k)\}}. \tag{3.4}$$

In the following, we will employ the continuation theorem to prove the existence of positive ω-periodic solution to Eq. (3.4), which implies the existence of positive ω-periodic solution of Eq. (3.2). We first define $X = Y = l^{\omega}$ and

$$(Lx)(k) = x(k+1) - x(k), \quad (\mathbb{N}x)(k) = -p(k) + \frac{q(k)}{r + \exp\{nx(k)\}}$$

for any $x(k) \in X, k \in \mathbb{Z}$. It is easy to verify that L is a bounded linear operator and

$$\operatorname{Ker} L = l_c^{\omega}, \quad \operatorname{Im} L = l_0^{\omega}$$

as well as

$$\dim\ \operatorname{Ker} L = 1 = \operatorname{codim} \operatorname{Im} L,$$

Therefore, L is a Fredholm mapping of index zero.

Define

$$Px = \frac{1}{\omega}\sum_{s=0}^{\omega-1} x(s), \quad x \in X, \quad Qy = \frac{1}{\omega}\sum_{s=0}^{\omega-1} y(s), \quad y \in Y.$$

It is not difficult to show that P and Q are continuous projectors and

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q).$$

Furthermore, the generalized inverse (to L) $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_p(y) = \sum_{s=0}^{\omega-1} y(s) - \frac{1}{\omega}\sum_{s=0}^{\omega-1} (\omega - s)y(s).$$

Obviously, $Q\mathbb{N}$ and $K_p(I-Q)\mathbb{N}$ are continuous. Since X is a finite dimensional Banach spaces, from the Arzela–Ascoli theorem it follows that $K_p(I-Q)\mathbb{N}(\overline{\Omega})$ are compact for any open bounded set $\Omega \subset X$. Moreover, $Q\mathbb{N}(\bar{\Omega})$ is bounded. Hence, $\mathbb{N}$ is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$.

In the following, we shall find an appropriate open, bounded subset Ω for the application of the continuation theorem. Corresponding to the operator equation $Lx = \lambda\mathbb{N}x$, $\lambda \in (0, 1)$, we have

$$x(k+1) - x(k) = \lambda\left(-p(k) + \frac{q(k)}{r + \exp\{nx(k)\}}\right). \tag{3.5}$$

Summing on both sides of (3.5) from 0 to $\omega - 1$ with respect to k, we obtain

$$\overline{p}\omega = \sum_{0}^{\omega-1} \frac{q(k)}{r + \exp\{nx(k)\}}, \tag{3.6}$$

which implies, together with (3.3), that

$$x(\xi) \geqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}, \quad x(\eta) \leqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\}. \tag{3.7}$$

From (3.5) and (3.6), we have

$$\sum_{0}^{\omega-1} |x(k+1) - x(k)| \leqslant \sum_{0}^{\omega-1}\left(p(k) + \frac{q(k)}{r + \exp\{nx(k)\}}\right) = 2\overline{p}\omega. \tag{3.8}$$

It follows from Lemma 3.1 and (3.7) and (3.8) that

$$x(k) \leqslant x(\eta) + \sum_{0}^{\omega-1} |x(k+1) - x(k)| \leqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\} + 2\overline{p}\omega = M_1$$

and

$$x(k) \geqslant x(\xi) - \sum_{0}^{\omega-1} |x(k+1) - x(k)| \geqslant \frac{1}{n}\ln\left\{\frac{\overline{q}}{\overline{p}} - r\right\} - 2\overline{p}\omega = M_2.$$

Consequently,

$$|x(k)| \leqslant \max\{|M_1|, |M_2|\} = B_1. \tag{3.9}$$

Clearly, B_1 is independent of λ. Denote $B = B_1 + B_2$, where B_2 is taken sufficiently large such that $B_2 > \frac{1}{n}|\ln\{\frac{\overline{q}}{\overline{p}} - r\}|$. Let

$$\Omega = \{x(t) \in X : \|x\| < B\}.$$

It is clear that Ω is an open bounded set in X and verifies requirement (a) in Lemma 2.1. Furthermore, when $x \in \partial\Omega \cap \operatorname{Ker} L$, x is a constant with $\|x\| = B$. Thus,

$$Q\mathbb{N}x = -\overline{p}\omega + \frac{\overline{q}\omega}{r + \exp\{nx\}} \neq 0.$$

Further, it is not difficult to verify that

$$\deg\{JQ\mathbb{N}x, \omega \cap \operatorname{Ker} L, 0\} \neq 0,$$

where J can be the identity mapping since $\operatorname{Im} P = \operatorname{Ker} L$. By now we know that Ω verifies all the requirements of Lemma 2.1 and then Eq. (3.4) has at least one ω-periodic solution $x^*(k)$. Finally, let $y^*(k) = \exp\{x^*(k)\}$, then $y^*(k)$ is a positive ω-periodic solution of Eq. (3.2). The proof is complete. □

Acknowledgements

We are grateful to thank the anonymous referees for their careful reading and constructive suggestions which lead to truly significant improvement of the manuscript.

References

[1] R.P. Agarwal, Difference Equations and Inequalities: Theory, Methods and Applications, Marcel Dekker, New York, 2000.
[2] M. Fan, S. Agarwal, Periodic solutions of nonautonomous discrete predator–prey system of Lotka–Volterra type, Appl. Anal. 81 (2002) 801–812.
[3] M. Fan, K. Wang, Periodic solutions of a discrete time nonautonomous ratio-dependent predator–prey system, Math. Comput. Modelling 35 (2002) 951–961.
[4] R.E. Gaines, J.L. Mawhin, Coincidence Degree and Non-linear Differential Equations, Springer, Berlin, 1997.
[5] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, Academic Press, New York, 1993.
[6] J.D. Murry, Mathematical Biology, Springer, New York, 1989.
[7] V.G. Nazarenko, Influence of delay on auto-oscillation in cell populations, Biofisika 21 (1976) 352–356.
[8] S.H. Saker, Oscillation and stability in nonlinear delay differential equations of population dynamics, Math. Comp. Modelling 35 (2002) 295–301.
[9] S.H. Saker, Oscillation and stability in nonlinear delay differential equations of population dynamics, Appl. Anal. 81 (2002) 787–799.
[10] Y. Song, J. Wei, M. Han, Local and global Hopf bifurcation in a delayed hematopoiesis model, Internat. J. Bifur. Chaos Appl. Sci. Engrg. 14 (2004) 3909–3919.
[11] R. Zhang, Z. Wang, Y. Chen, Periodic solutions of a single species discrete population model with periodic harvest/stock, Comput. Math. Appl. 39 (2000) 77–90.